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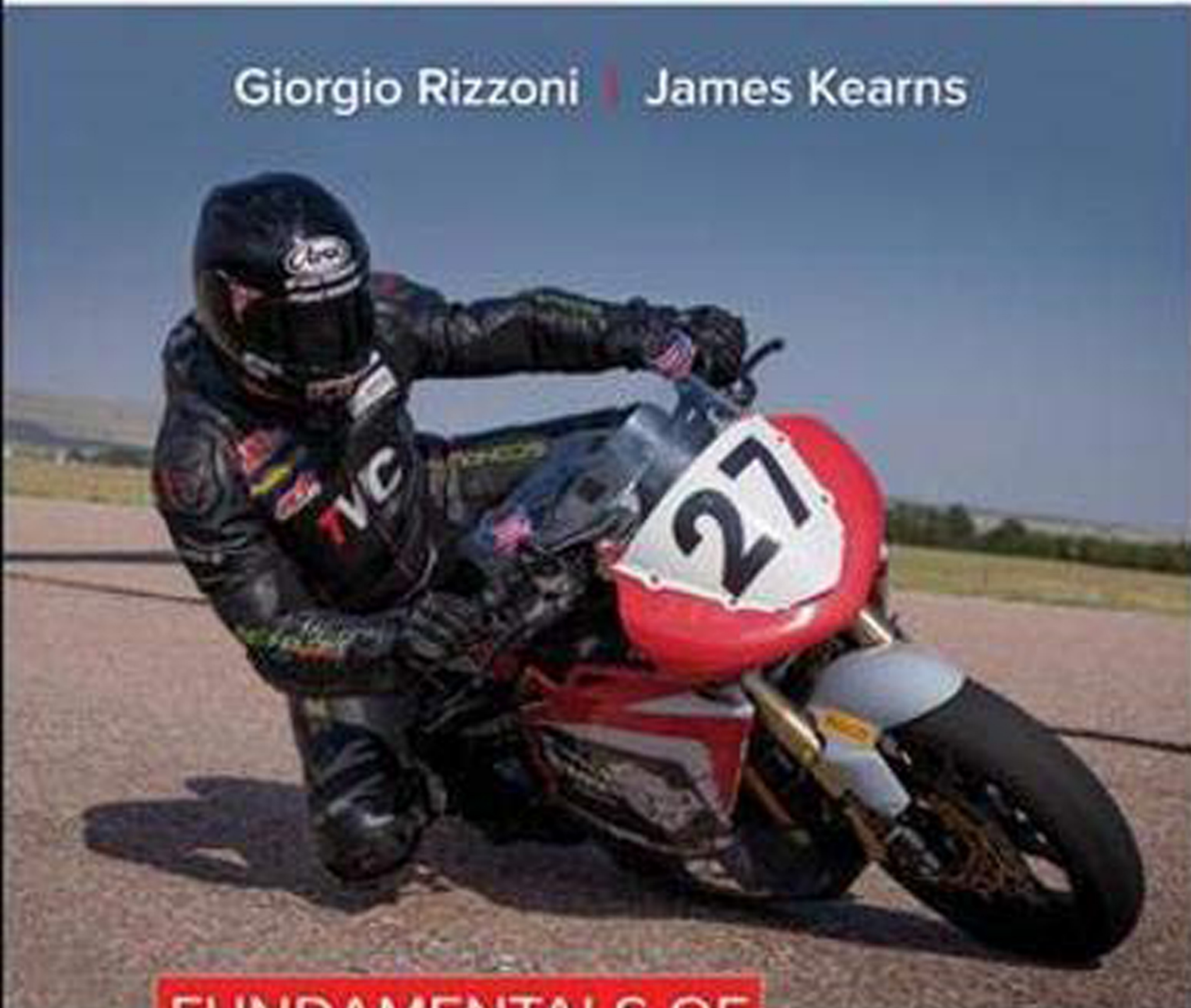
Giorgio Rizzoni | James Kearns



FUNDAMENTALS OF ELECTRICAL ENGINEERING

2e

McGraw Hill

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FUNDAMENTALS OF ELECTRICAL ENGINEERING

Second Edition

Giorgio Rizzoni
The Ohio State University

James Kearns
York College of Pennsylvania

Mc Graw Hill

FUNDAMENTALS OF ELECTRICAL ENGINEERING

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To our families

About the Authors

Giorgio Rizzoni, the *Ford Motor Company Chair in ElectroMechanical Systems*, is a Professor of Mechanical and Aerospace Engineering and of Electrical and Computer Engineering at The Ohio State University (OSU). He received his B.S. in 1980, his M.S. in 1982, and his Ph.D. in 1986, in Electrical and Computer Engineering, all from the University of Michigan. Since 1999 he has been the director of the Ohio State University Center for Automotive Research (CAR), an interdisciplinary research center in the OSU College of Engineering.

Dr. Rizzoni's research interests are in the dynamics and control of future ground vehicle propulsion systems, including advanced engines, alternative fuels, electric and hybrid-electric drivetrains, energy storage systems, and fuel cell systems. He has contributed to the development of a graduate curriculum in these areas and has served as the director of three U.S. Department of Energy Graduate Automotive Technology Education Centers of Excellence: *Hybrid Drivetrains and Control Systems* (1998–2004), *Advanced Propulsion Systems* (2005–2011), and *Energy Efficient Vehicles for Sustainable Mobility* (2011–2016).

In 1999 Dr. Rizzoni established an automotive industry research consortium that today sees the participation of over 20 automotive OEMs and suppliers; in 2008 he created the SMART@CAR consortium, focusing on plug-in hybrid and electric vehicles and vehicle-grid interaction, with funding from electric utilities, automotive OEMS, and electronics suppliers. Through the Ohio Third Frontier Wright Project Program he created a *Center of Excellence for Commercial Hybrid Vehicles* in 2009, and a *Center of Excellence for Energy Storage Technology* in 2010.

Dr. Rizzoni is a Fellow of IEEE (2004), a Fellow of SAE (2005), a recipient of the 1991 National Science Foundation Presidential Young Investigator Award, and of several other technical and teaching awards.

The OSU Center for Automotive Research

The OSU Center for Automotive Research, CAR, is an interdisciplinary research center in the OSU College of Engineering founded in 1991 and located in a 50,000 ft^2 building complex on the west campus of OSU. CAR conducts interdisciplinary research in collaboration with the OSU colleges of Engineering, Medicine, Business, and Arts and Sciences, and with industry and government partners. CAR research aims to: develop efficient vehicle propulsion and energy storage systems; develop new sustainable mobility concepts; reduce the impact of vehicles on the environment; improve vehicle safety and reduce occupant and pedestrian injuries; increase vehicle autonomy and intelligence; and create quieter and more comfortable automobiles. A team of 50 administrative and research staff supports some 40 faculty, 120 graduate and 300 undergraduate students and maintains and makes use of advanced experimental facilities. Dr. Rizzoni has led CAR for two decades, growing its research expenditures from $1M per year to over $15M today, and engaging CAR in a broad range of technology commercialization activities, start-up company incubation and spin-out, as well as providing a broad range of engineering services to the automotive industry.

CAR is also the home of the OSU Motorsports program, which supports the activities of five student vehicle competition programs of several student vehicle competition programs including: the Buckeye Bullet (holder of all current U.S. and FIA electric vehicle land speed records), the EcoCAR hybrid-electric vehicle team, the Formula Buckeyes and Baja Buckeyes SAE teams, and the Buckeye Current electric motorcycle racing team.

Jim Kearns is an Associate Professor of Electrical & Computer Engineering at York College of Pennsylvania. He received a B.S. in Mechanical Engineering (SEAS) and a B.S. in Economics (Wharton) from the University of Pennsylvania in 1982. Subsequently, he received his M.E. from Carnegie-Mellon University in 1984, and his Ph.D. from the Georgia Institute of Technology in 1990, both in Mechanical

Engineering. While at Georgia Tech he was the recipient of a Presidential Fellowship. Subsequently, he worked as a Postdoctoral Fellow at the Applied Research Laboratory of the University of Texas—Austin.

In 1992, Dr. Kearns took his first teaching position at the Universidad del Turabo in Gurabo, Puerto Rico, where he worked with a small group of faculty and staff to build and develop a new school of mechanical engineering. In addition to other duties, he was tasked with developing a curriculum on electromechanics. During this time Dr. Kearns spent his summers at Sandia National Laboratories as a University Fellow.

In 1996, Dr. Kearns was the second full-time engineering faculty member hired by York College of Pennsylvania to (once again) develop a new mechanical engineering program with an emphasis on Mechatronics. As a result of that work, Jim was asked in 2003 to develop a new electrical and computer engineering program at YCP. Jim served as program coordinator until July 2010.

Throughout Dr. Kearns professional career he has been involved in teaching and research related to physical acoustics and electromechanical systems. His interest in electrical engineering began during his Ph.D. studies, when he built spark generators, DC power supplies, and signal amplifiers for his experiments in physical acoustics. His steady pursuit of electromechanical engineering education has been the hallmark of his professional career. Dr. Kearns has been involved in a variety of pedagogical activities, including the development and refinement of techniques in electrical engineering education, and recently, the application of Thévenin’s theorem to determine characteristic parameters of electrical networks.

Dr. Kearns is a member of IEEE and ASEE. He is active in faculty governance at York College, where he is a past chair of its Tenure and Promotion committee and its Student Welfare committee. Dr. Kearns recently completed a four-year term as Vice-President and then President of the York College Academic Senate.

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About the Cover

The Buckeye Current electric race motorcycle is an award-winning collegiate motorcycle racing team founded in 2010 by two students with a big dream. Over the course of the team's 10-year history, hundreds of students have had the opportunity to help design and race electric race motorcycles. Based at The Ohio State University's Center for Automotive Research, the team pushes the limits of electric vehicles by developing motorcycles that have raced, and won, across the world. The Buckeye Current team has set speed records and has raced at the Isle of Man and Pike's Peak. Go to the website http://org.osu.edu/buckeyecurrent for more information.

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Brief Contents

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Preface

The pervasive presence of electronic devices and instrumentation in all aspects of engineering design and analysis is one of the manifestations of the electronic revolution that has characterized the last sixty years. Every aspect of engineering practice, and of everyday life, has been affected in some way or another by electrical and electronic devices and instruments. Laptop and tablet computers along with so-called “smart” phones, and touchscreen interfaces are perhaps the most obvious manifestations. These devices, and their underlying technology, have brought about a revolution in computing, communication, and entertainment. They allow us to store, process, and share professional and personal data and to access audio (most notably, music) and video of every variety. These advances in electrical engineering technology have had enormous impacts on all other fields of engineering, including mechanical, industrial, computer, civil, aeronautical, aerospace, chemical, nuclear, materials, and biological engineering. This rapidly expanding electrical and electronic technology has been adopted, leveraged, and incorporated in engineering designs across all fields. As a result, engineers work on projects requiring effective communication across multiple disciplines, one of which is nearly always electrical engineering.

0.1 OBJECTIVES

Engineering education and professional practice continue to undergo profound changes in an attempt to best utilize relevant advances in electronic technology. The need for textbooks and other learning resources that relate these advances to engineering disciplines beyond electrical and computer engineering continues to grow. This fact is evident in the ever-expanding application and integration of electronics and computer technologies in commercial products and processes. This textbook and its associated learning resources represent one effort to make the principles of electrical and computer engineering accessible to students in various engineering disciplines.

The principal objective of the book is to present the *fundamentals* of electrical, electronic, and electromechanical engineering to an audience of engineering majors enrolled in introductory and more advanced or specialized electrical engineering courses.

A second objective is to present these *fundamentals* with a focus on important results and common yet effective *analytical and computational tools* to solve practical problems.

Finally, a third objective of the book is to illustrate, by way of concrete, fully worked examples, a number of relevant *applications* of electrical engineering. These examples are drawn from the authors' industrial research experience and from ideas contributed by practicing engineers and industrial partners.

These three objectives are met through the use of various pedagogical features and methods.

0.2 ORGANIZATION

The second edition contains several significant organizational changes. However, the substance of the book, while updated, is essentially unchanged. The most obvious organizational change is the location of example problems within each chapter. In the previous edition, examples were mixed in with the text so that students would encounter examples immediately after each key concept. While this type of Page xviorganization works well for a first read, it has the disadvantage of making example problems difficult to locate for review. Since it is critical that students be able to easily and efficiently locate example problems when preparing for exams, in this edition of the book, with few exceptions, all example problems have been placed at the end of each section within a chapter.

A continued and enhanced emphasis on problem solving can be found in this edition. All the highlighted *Focus on Methodology* boxes found in the first edition were renamed *Focus on Problem Solving*, and many of them were rewritten to clarify and add additional detail to the steps needed by students to successfully complete end-of-chapter homework problems.

An effort was also made to reduce the aesthetic complexity of the book, without sacrificing technical content or overall aesthetic appeal. We believe that effective reading is promoted by less clutter and visual "noise," if you will. For example, a careful comparison of the first and second editions will reveal our effort to produce cleaner and sharper figures that retain only that information relevant to the issue or problem being discussed.

In addition, a thorough, exhaustive, page-by-page search was made to locate errors in the text, equations, figures, references to equations and figures, examples, and homework problems. Speaking of homework problems, the second edition contains 861 homework problems, of which over 300 are new to this edition, and, where necessary and appropriate, example problems were updated.

The book remains divided into three major parts:

I. **Circuits**

II. **Electronics**

III. **Electromechanics**

The pedagogical enhancements made within each part are discussed below.

0.3 PEDAGOGY AND CONTENT

Part I: Circuits

The first part of the book has undergone major revision from the first edition.

Chapter 1 begins with an emphasis on developing a student's ability to recognize structure within a circuit diagram. It is the authors' experience that this ability is key to student success. Yet, many books contain little content on developing this ability; the result is that many students wander into more difficult topics still viewing a circuit as simply an unruly collection of wires and elements.

The approach taken in this book is to encourage students to initially *focus on nodes*, rather than elements, in a circuit. For example, some of the earliest exercises in this book simply ask students to count the number of nodes in a circuit diagram. One immediate advantage of this patient approach is that it teaches students to disregard the particular aesthetic structure shown in a circuit diagram and instead to recognize and focus on the technical structure and content.

Methods of Problem Solving were enhanced and clarified. Throughout these chapters students are encouraged to think of problem solving in two steps: first **simplify**; then **solve**. In addition to being an effective problem-solving method, this method provides context for the power and importance of equivalent circuits, in general, and Thévenin's theorem, in particular. In the chapters on transient analysis Page xviiand frequency response, foundational first- and second-order circuit *archetypes* are identified. Students are encouraged to simplify, when possible, transient circuit problems to these archetypes. In effect, they become clear targets for students when problem solving. Thévenin's and Norton's theorems and the principle of superposition are used throughout these chapters to simplify complicated circuits to the archetypes.

Finally, a greater emphasis was placed on visualizing phasors in the complex plane and understanding the key role of the unit phasor and Euler's theorem. Throughout the chapters on AC circuits and power students are encouraged to focus on the concepts of impedance and power triangles, and their similarity.

Part II: Electronics

While much of the content on electronics in Part II is unchanged from the first edition, the problem-solving strategies and techniques for transistor circuits were enhanced and clarified. The focus on simple but useful circuit examples was not changed.

Similar to the approach taken in Part I, Chapter 7 on operational amplifiers emphasizes three *amplifier archetypes* (the unity-gain buffer, the inverting amplifier, and the non-inverting amplifier) before introducing variations and applications.

The emphasis in Chapters 9 and 10 on large-signal models of BJTs and FETs and their applications was retained; however, an appropriate, but limited, presentation of small-signal models was included to support the discussion of AC amplifiers. These two chapters present an uncomplicated and practical treatment of the analysis and design of simple amplifiers and switching circuits using large-signal models.

Chapter 11 presents an overview of combinational and sequential logic modules, providing a comprehensive overview of digital logic circuits.

Part III: Electromechanics

Part III on electromechanics has been revised for accuracy and pedagogy, but its contents are largely unchanged. This part has been used by the first author for many years as a supplement in a junior-year System Dynamics course for mechanical engineers.

0.4 NOTATION

The notation used in this book for various symbols (variables, parameters, and units) has been updated but still follows generally accepted conventions. Distinctions in notation can be subtle. Luckily, very often the context in which a symbol appears makes its meaning clear. When the meaning of a symbol is not clear from its context a correct reading of the notation is important. A reasonably complete listing of the symbols used in this book and their notation is presented below.

For example, an uppercase roman font is used for units such as volts (V) and amperes (A). An uppercase italics math font is used for real parameters and variables such as resistance (R) and DC voltage (V). Notice the difference between the variable V and the unit V. Further, an uppercase bold math font is used for complex quantities such as voltage and current phasors ($\mathbf{V}$ and $\mathbf{I}$) as well as impedance ($\mathbf{Z}$), conductance ($\mathbf{Y}$), and frequency response functions ($\mathbf{H}$ and $\mathbf{G}$). Lowercase italic Page xviisymbols are, in general, time dependent variables, such as voltage (v or $v(t)$) and current (i or $i(t)$), where (t) is an explicit indication of time dependence. Lowercase italic variables may represent constants in specific cases. Uppercase italic variables are reserved for constant (time-invariant) values exclusively.

Various subscripts are also used to denote particular instances or multiple occurrences of parameters and variables. Exponents are italicized superscripts.

Finally, in electrical engineering the imaginary unit $\sqrt{-1}$ is always represented by j rather than i, which is used by mathematicians. The reason for the use of j instead of i should be obvious!

Quantity	Symbol	Description
Voltage	v or $v(t)$	Time Dependent and Real
	V	Time Invariant and Real
	$\mathbf{V}$	Complex Phasor
Effective (rms) voltage	$\tilde{V}$	Time Invariant and Real
Current	i or $i(t)$	Time Dependent and Real
	I	Time Invariant and Real
	$\mathbf{I}$	Complex Phasor
Effective (rms) current	$\tilde{I}$	Time Invariant and Real
Volts	V	Unit of voltage
Amperes	A	Unit of current
Resistance	R	Real
Inductance	L	Real
Capacitance	C	Real
Reactance	X	Frequency Dependent and Real
Impedance	$\mathbf{Z}$	Frequency Dependent and Complex
Conductance	$\mathbf{Y}$	Frequency Dependent and Complex
Transfer Function	$\mathbf{G}$ or $\mathbf{H}$	Frequency Dependent and Complex
Cyclical Frequency	f	Time Invariant and Real
Angular Frequency	ω	Time Invariant and Real
Angle	θ	Time Invariant and Real
Amplitude	A	Time Invariant and Real

0.5 SYSTEM OF UNITS

This book employs the International System of Units (also called SI, from the French Système International des Unitès). SI units are adhered to by virtually all professional engineering societies and are based upon the seven fundamental quantities listed in Table 0.1. All other units are derived from these base units. An example of a derived unit is the radian, which is a measure of plane angles. In this book, angles are in units of radians unless explicitly given otherwise as degrees.

Since quantities often need to be described in large multiples or small fractions of a unit, the standard prefixes listed in Table 0.2 are used to denote SI units in powers of 10. In general, engineering units are expressed in powers of 10 that are multiples of 3. For example, 10^{-4} s would be expressed as 100×10^{-6} s, or 100 µs.

Tables 0.1 and 0.2 are useful references when reading this book.

Table 0.1 SI units

Quantity	**Unit**	**Symbol**
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Substance	Mole	mol
Luminous intensity	Candela	cd

Table 0.2 Standard prefixes

Prefix	**Symbol**	**Power**
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	µ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

0.6 FEATURES OF THE SECOND EDITION

Pedagogy

The second edition continues to offer all the time-tested pedagogical features available in the earlier editions.

- **Learning Objectives** offer an overview of key chapter ideas. Each chapter opens with a list of major objectives, and throughout the chapter the learning objective icon indicates targeted references to each objective.
- **Focus on Problem Solving** sections summarize important methods and procedures for the solution of common problems and assist the student in developing a methodical approach to problem solving.
- **Clearly Illustrated Examples** illustrate relevant applications of electrical engineering principles. The examples are fully integrated with the Focus on Problem Solving material, and each one is organized according to a prescribed set of logical steps.
- **Check Your Understanding** exercises follow each set of examples and allow students to confirm their mastery of concepts.
- **Make the Connection** sidebars present analogies that illuminate electrical engineering concepts using other concepts from engineering disciplines.
- **Focus on Measurements** boxes emphasize the great relevance of electrical engineering to the science and practice of measurement.

Instructor Resources on Connect:

Instructors have access to these files, which are housed in Connect.

- **PowerPoint presentation slides** of important figures from the text
- **Instructor's Solutions Manual** with complete solutions

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Writing Assignment

Available within McGraw Hill Connect®, the Writing Assignment tool delivers a learning experience to help students improve their written communication skills and conceptual understanding. As an instructor you can assign, monitor, grade, and provide feedback on writing more efficiently and effectively.

Remote Proctoring & Browser-Locking Capabilities

connect + proctorio

New remote proctoring and browser-locking capabilities, hosted by Proctorio within Connect, provide control of the assessment environment by enabling security options and verifying the identity of the student.

Seamlessly integrated within Connect, these services allow instructors to control students' assessment experience by restricting browser activity, recording students' activity, and verifying students are doing their own work.

Instant and detailed reporting gives instructors an at-a-glance view of potential academic integrity concerns, thereby avoiding personal bias and supporting evidence-based claims.

0.7 ACKNOWLEDGMENTS

The authors would like to recognize the help and assistance of reviewers, student's, and colleagues who have provided invaluable support. In particular, Dr. Ralph Tanner of Western Michigan University has painstakingly reviewed the book for accuracy and has provided rigorous feedback, and Ms. Jiyu Zhang, PhD student at Ohio State, has been generous in her assistance with the electromechanical systems portion of the chapter. Dr. Isabel Fernandez Puentes of The Ohio State University has reviewed the final manuscript for accuracy, with great attention to detail. The authors are especially grateful to Dr. Domenico Bianchi and Dr. Gian Luca Storti for creating many new homework problems and solutions and for their willingness to pitch in whenever needed. This second edition is much improved due to their efforts. Finally, Mr. Riccardo Palomba has painstakingly assembled the Instructor's Solutions Manual.

Throughout the preparation of this edition, Kathryn Rizzoni has provided editorial support and has served as an interface to the editorial staff at MHHE. We are grateful for her patience, her time invested in the project, her unwavering encouragement, her kind words, and her willingness to discuss gardening and honeybees.

The book has been critically reviewed by:

- Riadh Habash—The University of Ottawa
- Ahmad Nafisi—California Polytechnic State University
- Raveendra Rao—The University of Western Ontario
- Belinda Wang—The University of Toronto
- Brian Peterson—United States Air Force Academy
- John Durkin—University of Akron
- Chris Klein—Ohio State University
- Ting-Chung Poon—Virginia Tech
- James R. Rowland—University of KansasPage xxi
- N. Jill Schoof—Maine Maritime Academy
- Shiva Kumar—McMaster University
- Tom Sullivan—Carnegie Mellon University
- Dr. Kala Meah—York College of Pennsylvania

In addition, we would like to thank the many colleagues who have pointed out errors and inconsistencies and who have made other valuable suggestions.

Comments by Giorgio Rizzoni

As always, a new edition represents a new era. I am truly grateful to my friend and co-author, Jim Kearns, for taking on a new challenge and for bringing his perspective and experience to the book. Jim and I share a passion for teaching, and throughout this project we have invariably agreed on which course to take. It is not easy to find a suitable co-author in the life of a project of this magnitude, and I have been fortunate to find a friend willing to undertake a new journey with me.

The years go by, but my family continues to be an endless source of joy, pleasant surprises and, always, smiles. Many thanks to Kathryn, Alex, Cat, and Michael for always being there to support and encourage me.

Comments by James Kearns

My association with this remarkable book continues to be a great privilege and honor. Its contents continue to reflect the enormous effort and expertise of the principal author, and my dear friend, Dr. Giorgio Rizzoni. His leadership and vision were essential to the creation of this new edition. I remain awestruck by his seemingly unbounded energy and enthusiasm and humbled by his kind, considerate, and generous ways.

As with all things, the love and support of my family and friends sustained me throughout this work. My children, Kevin, Claire, and Caroline, continue to bless, inspire, and inform my daily life.

Finally, I wish to once again thank my parents for their many years of unconditional love and support. Despite having lost both of them in recent years they remain very much alive within me and present in my work. Can anyone ever begin to measure or repay the gift of loving parents?

Guided Tour

Learning Objectives offer an overview of key chapter ideas. Each chapter opens with a list of major objectives, and throughout the chapter the learning objective icon indicates targeted references to each objective.

> steady state of any variable?
> 4. How fast or slow is that transition?
> 5. What is the final steady state of any variable?
>
> Two types of circuits are examined in this chapter: first-order *RC* and *RL* circuits, which contain a single storage element, and second-order circuits, which contain two irreducible storage elements. The simplest of the second-order circuits to analyze are the series *RLC* and parallel *RLC* circuits. Other more complicated second-order circuits exist, as do higher-order circuits; however, since all the fundamental behaviors of transient circuits are revealed in the types just mentioned, they are the focus of this chapter.
>
> A first-order circuit contains a single storage element. A second-order circuit contains two irreducible storage elements.
>
> Throughout this chapter, practical applications of first- and second-order circuits are introduced. Numerous analogies are presented to emphasize the general nature of the solution methods and their applicability to a wide range of physical systems, including hydraulics, mechanical systems, and thermal systems.
>
> **Learning Objectives**
>
> *Students will learn to...*
> 1. Understand the fundamental qualities of transient responses. *Section 4.1*
> 2. Write differential equations in standard form for circuits containing inductors and capacitors. *Section 4.2*
> 3. Determine the steady state of DC circuits containing inductors and capacitors. *Section 4.2*
> 4. Determine the complete solution of first-order circuits excited by switched DC sources. *Section 4.3*
> 5. Determine the complete solution of second-order circuits excited by switched DC sources. *Section 4.4*
> 6. Understand analogies between electric circuits and hydraulic, thermal, and mechanical systems. *Sections 4.1–4.4*

Focus on Problem Solving sections summarize important methods and procedures for the solution of common problems and assist the student in developing a methodical approach to problem solving.

FOCUS ON PROBLEM SOLVING

ROOTS OF SECOND-ORDER SYSTEMS

The general form of the roots s_1 and s_2 is
$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$.
The nature of these roots depends upon the argument of the square root.

Case 1: **Distinct, negative, real roots**. This case occurs when $\zeta > 1$ since the term under the square root sign is positive. The result is $s_{1,2} = -\omega_n\left[\zeta \pm \sqrt{\zeta^2 - 1}\right]$ and a second-order **overdamped response.**

Case 2: **Identical, negative, real roots**. This case occurs when $\zeta = 1$ since the term under the square root is zero. The result is a repeated root $s = -\zeta\omega_n = -\omega_n$ and a second-order **critically damped response.**

Case 3: **Complex conjugate roots**. This case holds when $\zeta < 1$ since the term under the square root is negative. The result is a pair of complex conjugate roots $s_{1,2} = -\omega_n\left[\zeta \pm j\sqrt{1 - \zeta^2}\right]$ and a second-order **underdamped response**.

Clearly illustrated examples present relevant applications of electrical engineering principles. The examples are fully integrated with the Focus on "Problem" Solving material, and each one is organized according to a prescribed set of logical steps.

EXAMPLE 4.15 Complete Response of Critically Damped Second-Order Circuit

Problem

Determine the complete response for the voltage v_C shown in Figure 4.45.

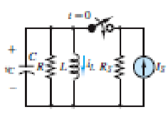


Figure 4.45 Circuit for Example 4.15.

Solution

Known Quantities: I_S; R; R_S; C; L.

Find: The complete response of the differential equation in v_C describing the circuit in Figure 4.45.

Schematics, Diagrams, Circuits, and Given Data: $I_S = 5$ A; $R = R_S = 500\ \Omega$; $C = 2\ \mu$F; $L = 500$ mH.

Assumptions: None.

Analysis:

Step 1: Steady-state response. With the switch open for a long time, any energy stored in the capacitor and inductor has had time to be dissipated by the resistor; thus, the currents and voltages in the circuit are zero: $i_L(0^-) = 0$, $v_C(0^-) = v(0^-) = 0$.

After the switch has been closed for a long time and all the transients have died, the capacitor becomes an open-circuit, and the inductor behaves as a short-circuit. With the inductor behaving as a short-circuit, all the source current will flow through the inductor, and $i_L(\infty) = I_S = 5$ A. On the other hand, the current through the resistor is zero, and therefore $v_C(\infty) = v(\infty) = 0$ V.

Step 2: Initial conditions. Two initial conditions are needed to solve a second-order circuit. These two initial conditions always rely on two continuity conditions: the current through an inductor and the voltage across a capacitor are continuous. That is, $i_L(0^-) = i_L(0^+) = 0$ A and $v_C(0^-) = v_C(0^+) = 0$ V. Since the differential equation is in the variable v_C, the two needed initial conditions are $v_C(0^+)$ and $dv_C(0^+)/dt$. These can be found by applying KCL at $t = 0^+$:

$$I_S - \frac{v_C(0^+)}{R_S} - i_L(0^+) - \frac{v_C(0^+)}{R} - C\frac{dv_C(0^+)}{dt} = 0$$

Since $v_C(0^+) = 0$ and $i_L(0^+) = 0$, the result is: $dv_C(0^+)/dt$:

$$\frac{dv_C(0^+)}{dt} = \frac{I_S}{C} = \frac{5}{2\times 10^{-6}} = 2.5\times 10^6\frac{\text{V}}{\text{s}}$$

Step 3: Differential equation. Apply KCL at the upper node to find a first-order differential equation in the two state variables v_C and i_L:

$$I_S - \frac{v_C}{R_S} - i_L - \frac{v_C}{R} - C\frac{dv_C}{dt} = 0 \qquad t \geq 0$$

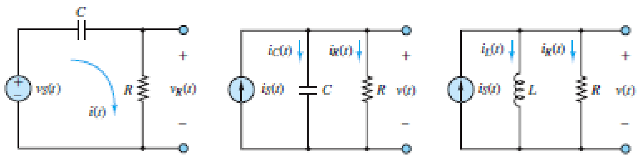
Differentiate both sides to obtain a second-order differential equation. Then, note that v_C is also the voltage across the inductor such that the constitutive i-v relation for the inductor can be written as:

$$v_C = v_L = L\frac{di_L}{dt}$$

Check Your Understanding exercises follow each set of examples and allow students to confirm their mastery of concepts.

CHECK YOUR UNDERSTANDING

Use the differential i-v relations for capacitors and inductors along with KVL or KCL to write the differential equation for each of the circuits shown below.



(a) (b) (c)

Answer (a) $RC\frac{dv_C(t)}{dt} + v_C(t) = v_S(t)$; (b) $RC\frac{dv(t)}{dt} + v(t) = Ri_S(t)$;
(c) $\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = i_S(t)$

Make the Connection sidebars present analogies that illuminate electrical engineering concepts using concepts form other engineering disciplines.

Second-order circuit response is more complex and described by one of three possible cases, each of which is determined by a parameter ζ known as the *dimensionless damping ratio*, as explained in detail in Section 4.4. When $\zeta > 1$, the response is said to be *overdamped* and is the sum of two first-order decaying exponentials, each with its own distinct time constant. When $\zeta = 1$, the response is said to be *critically damped*. When $\zeta < 1$, the response is said to be *underdamped*. As shown in Figure 4.5, the responses in these latter two cases *cannot* be simply described by two decaying exponentials.

Long-Term Steady State

The long-term steady state is that which remains after the transient response has decayed completely. For the first-order decaying exponential shown in Figure 4.7 the long-term steady state is $x(\infty)$. The long-term steady state depends upon the independent sources present in the $t > 0$ circuit and is commonly expressed in terms of a *gain* K multiplied by a forcing function $F(t)$ that represents the contributions of those sources. For simplicity, only circuits with DC independent sources are considered in this chapter, with the result that only DC long-term steady states occur.

Complete Response

The complete response is simply the sum of the transient response and the long-term steady state. In general, the transient response will contain one unknown constant for each state variable in the circuit. Thus, the complete response will also contain the same number of unknown constants. The values of these unknown constants are determined by the initial conditions on the circuit at $t = 0^+$.

A common mistake when learning to solve transient circuit problems is to apply the initial conditions to the transient response alone rather than to the complete solution. Forewarned, forearmed; don't make this error!

Natural and Forced Responses

Often, it is useful to express the complete response as the sum of *natural* and *forced* responses instead of the sum of a transient response and long-term steady state. Either way the complete response is unchanged. The natural response is that part of the complete system response due to the initial energy stored in the system at $t = 0$. The forced response is that part due to independent sources present in the $t > 0$ circuit.

As will be shown in the following section, equation 4.9 expresses the complete response $x(t)$ of an arbitrary first-order circuit variable as the sum of a transient response, with its characteristic exponential decay, and a long-term steady state $x(\infty)$.

$$x(t) = [x(0^+) - x(\infty)]e^{-t/\tau} + x(\infty) \quad (4.9)$$

The transient response portion includes the difference between the initial condition $x(0^+)$ and the long-term steady state. This expression can be reconstructed as:

$$x(t) = x_N(t) + x_F(t) = x(0^+)e^{-t/\tau} + x(\infty)(1 - e^{-t/\tau}) \quad (4.10)$$

The first and second terms in equation 4.10 are known as the *natural* and *forced* responses, $x_N(t)$ and $x_F(t)$, respectively. A similar construction can be made for the complete response of a second-order circuit.

MAKE THE CONNECTION

Thermal System Dynamics

To describe the dynamics of a thermal system, we write a differential equation based on energy balance. The difference between the heat added to the mass by an external source and the heat leaving the same mass (by convection or conduction) must be equal to the heat stored in the mass:

$$q_{in} - q_{out} = q_{stored}$$

An object is internally heated at the rate q_{in} in ambient temperature $T = T_0$; the thermal capacitance and thermal resistance are C_t and R_t. From energy balance:

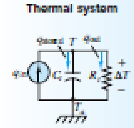
$$q_{in}(t) - \frac{T(t) - T_0}{R_t} = C_t\frac{dT(t)}{dt}$$

$$R_tC_t\frac{dT(t)}{dt} + T(t) = R_tq_{in}(t) + T_a$$

$$\tau_t = R_tC_t \quad K_{S} = R_t$$

This first-order system is identical in its form to an electric RC circuit, as shown below.

Thermal system



Equivalent electric circuit

Focus on Measurements boxes emphasize the great relevance of electrical engineering to the science and practice of measurement.

FOCUS ON MEASUREMENTS

Capacitive Displacement Transducer and Microphone

As shown in Figure 3.2, the capacitance of a flat parallel-plate capacitor is:

$$C = \frac{\varepsilon A}{d} = \frac{\kappa \varepsilon_0 A}{d}$$

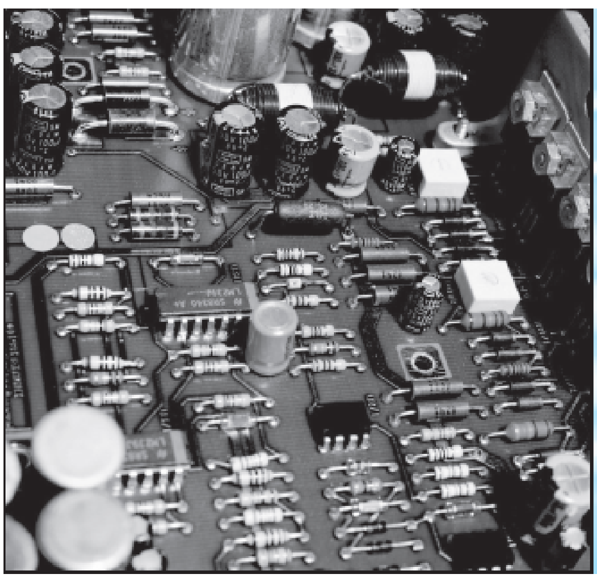
where ε is the **permittivity** of the dielectric material, κ is the dielectric constant, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m is the permittivity of a vacuum, A is the area of each of the plates, and d is their separation. The dielectric constant for air is $\kappa_{air} \approx 1$. Thus, the capacitance of two flat parallel plates of area 1 m^2, separated by a 1-mm air gap, is 8.854 nF, a very small value for such large plates. As a result, flat parallel-plate capacitors are impractical for use in most electronic devices. On the other hand, parallel-plate capacitors find application as motion transducers, that is, as devices that can measure the motion or displacement of an object. In a capacitive motion transducer, the plates are designed to allow relative motion when subjected to an external force. Using the capacitance value just derived for a parallel-plate capacitor, one can obtain the expression

$$C = \frac{8{,}854 \times 10^{-3} A}{x}$$

where C is the capacitance in picofarads, A is the area of the plates in square millimeters, and x is the separation distance in millimeters. Note that the change in C due to a change in x is nonlinear, since $C \propto 1/x$. However, for small changes in x, the change in C is

FUNDAMENTALS OF ELECTRICAL ENGINEERING

PART I
CIRCUITS



Arthur S. Aubry/Stockbyte/Getty Images

Page 3

C H A P T E R 1

FUNDAMENTALS OF ELECTRIC CIRCUITS

Chapter 1 is the foundation for the entire book and presents the fundamental laws that govern the behavior of electric circuits. Basic features and terminology of electric circuits, such as nodes, branches, meshes, and loops, are defined, and the three fundamental laws of circuit analysis. Kirchhoff's current and voltage laws and Ohm's law, are introduced. The concept of electric power and the passive sign convention are introduced along with basic circuit elements—sources and resistors. Basic analytic techniques—voltage and current division—are introduced along with some engineering applications. Examples include a description of strain gauges, circuits related to the measurement of force and other mechanical variables, and a study of an automotive throttle position sensor. A brief discussion of measurement instruments is also included. Finally, the chapter closes with a discussion of the source-load perspective and an application of it to find the same voltage and current division results obtained earlier in the chapter.Page 4

Learning Objectives

Students will learn to...

1. Identify the principal *features of electric circuits or networks*: nodes, loops, meshes, and branches. Sections 1.1.
2. Apply *Kirchhoff's laws* to simple electric circuits. Sections 1.2–1.3.
3. Apply the *passive sign convention* to compute the power consumed or supplied by circuit elements. Sections 1.4.
4. Identify *sources* and *resistors* and their *i-v characteristics*. Sections 1.5–1.6.
5. Apply *Ohm's law* and *voltage and current division* to calculate unknown voltages and currents in simple series, parallel, and series-parallel circuits. Sections 1.6–1.8.
6. Understand the impact of internal resistance in practical models of voltage and current sources as well as of voltmeters, ammeters, and wattmeters. Sections 1.9–1.10.

1.1 FEATURES OF NETWORKS AND CIRCUITS

A *network* can be defined as a collection of interconnected objects. In an electric network, *elements*, such as resistors, are connected by wires. An electric *circuit* can be defined as an electric network within which at least one closed path exists and around which electric charge may flow. All electric circuits are networks, but not all electric networks contain a circuit. In this book, a circuit is any network that contains at least one complete and closed path.

There are two principal quantities within a circuit: current and voltage. *The primary objective of circuit analysis is to determine one or more unknown currents and voltages.* Once these currents and voltages are determined, any other aspect of the circuit, such as its power requirements, efficiency, and speed of response, can be computed.

Two useful concepts for circuit analysis are those of a source and of a load. In general, the load is the circuit element or segment of interest to the designer or user of the circuit. By default, the source is everything else not included in the load. Typically, the source provides energy and the load consumes it for some purpose. For example, consider the simple physical circuit of a headlight attached to a car battery as shown in Figure 1.1(a). For the driver of the car, the headlight may be the circuit element of interest since it enables the driver to see the road at night. From this perspective, the headlight is the load and the battery is the source as shown in Figure

1.1(b), which is intuitively appealing because power flows from the source (the battery) to the load (the headlight). However, in general, it is not required nor necessarily true that power flows in this manner. Electric power is discussed later in this chapter.

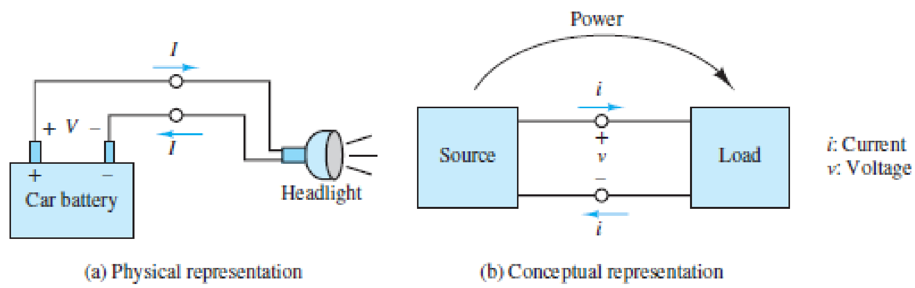


Figure 1.1 (a) Physical, and (b) conceptual representations of an electrical system

The use of the term *source* can be confusing at times because, as will be discussed later in this chapter, there are circuit elements known as *ideal voltage and current sources*, which have well-defined attributes and circuit symbols. These ideal sources, along with other circuit elements, are often the constituents of the source portion of a circuit, as well as the load portion. In this book, ideal sources are referred to as either voltage or current sources, explicitly, to avoid confusion.Page 5

Other key conceptual features of electric circuits are the *ideal wire*, *node*, *branch*, *loop*, and *mesh*. The concept of a node is particularly useful for correctly interpreting circuit diagrams. Many students struggle with circuit analysis simply because they lack an organizing perspective with which to interpret circuit diagrams. One particularly helpful perspective is to see electric circuits as comprised of elements situated between nodes. Once the concept of a node is well understood this perspective simplifies and clarifies many circuits that otherwise appear complicated.

Ideal Wire

Electric circuit and network *diagrams* are used to represent (approximately) actual electric circuits and networks. These diagrams contain *elements* connected by *ideal wires*. An ideal wire is able to conduct electric charge without any loss of electric potential. In other words, no work is required to move an electric charge along an ideal wire. Luckily, in many applications, actual wires are well approximated by ideal wires. However, there are applications where wiring accounts for significant losses of potential (e.g., long-distance transmission lines and microscopic integrated circuits). In these applications, the ideal wire approximation must be augmented and/or used with care. In this book, all wires in circuit and network diagrams are ideal, unless indicated otherwise.

Node

A **node** consists of one or more ideal wires connected together such that an electric charge can travel between any two points on the node without traversing a circuit element, such as a resistor. It is important to recognize that since a node consists of ideal wires only, every point on a node has the same electric potential, which is known as the *node voltage* and its value is relative to the other nodes in the network.

The *junction* of two or more ideal wires is often used to represent an entire node; however, it is important to recognize that a wire junction is not the entire node and that a node may contain multiple wire junctions. It is crucial to correctly identify and count nodes in the analysis of electric circuits. Figure 1.2 illustrates a helpful way to mark nodes. There are three nodes in Figure 1.2(a) and two nodes in Figure 1.2(b). It is sometimes convenient to use the concept of a **supernode,** which is simply a closed boundary enclosing two or more nodes, as shown in Figure 1.2(c). In the next section, you will learn that one of the two fundamental laws of network analysis, Kirchhoff's current law (KCL), is valid for any closed boundary; that is, it is valid for any node or supernode.Page 6

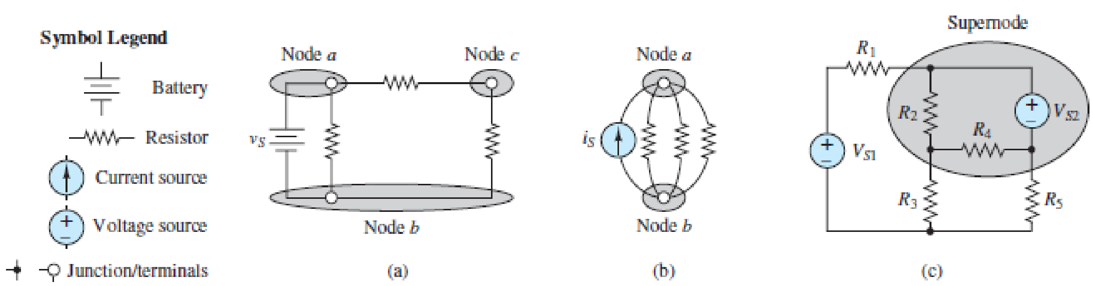


Figure 1.2 Illustrating nodes and supernodes in circuit diagrams

> Elements that sit between the same two nodes are said to be in *parallel* and have the same voltage across them.

It is also important to realize that since no work is required to move an electric charge along an ideal wire, the length and shape of an ideal wire has no impact on the behavior of a circuit. Likewise, since nodes are comprised of ideal wires, the extent and shape of a node has no impact on the behavior of a circuit. As a result, a node may be redrawn in any manner as long as the newly drawn node is attached to the same elements as the original node. Circuit diagrams are typically drawn, by convention, in a rectangular manner, with all wires drawn either side to side or up and down. However, many students find it helpful to redraw circuits so as to clarify the number and location of nodes in a circuit. Figure 1.3 shows two identical circuits drawn in two different ways. Can you tell that these circuits have the same number of nodes?

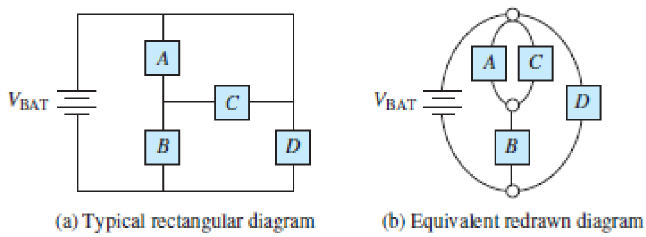


Figure 1.3 (a) A typical rectangular circuit diagram and (b) an equivalent redrawn diagram. A circuit can be redrawn to have almost any appearance; however, the nature of the circuit is unchanged if the number of nodes and the elements between those nodes remain unchanged.

Keep in mind that all forms of potential, including voltage, are relative quantities. For this reason, it is common to refer to the *change* in voltage *across* an element, or simply the voltage *across* an element. In circuit diagrams, a change in voltage across an element is indicated by the paired symbols + and −. Taken together as a single symbol they indicate the *assumed* direction of the change in voltage. However, as mentioned, it is also common to refer to a node voltage. To quantify a node voltage it is first necessary to select a *reference node.* Then, one can refer to the voltage of a node with the understanding that the value of that voltage is relative to the chosen reference node.Page 7

Any one node in a network can serve as the reference. The reference node and its value can be chosen freely, although a value of zero is usually chosen, for simplicity. It is often true that a smart choice of reference node will simplify the analysis that follows. A good rule of thumb is to select a node that is connected to a large number of elements.

A reference node is designated by the symbol shown in Figure 1.4(a). This symbol is also used to designate *earth ground* in applications. It is common for this symbol to appear multiple times in complicated circuits. Still, there is only one reference node per circuit. To reduce the apparent complexity of such circuits, multiple reference symbols are used to minimize the amount of displayed reference node wiring. It is simply understood that all nodes to which these symbols are attached are, in fact, connected by ideal wires and therefore part of one large reference node. Figure 1.4(b) and (c) illustrate this practice. The concepts of reference node, earth ground, and *chassis ground* are discussed later in this chapter.

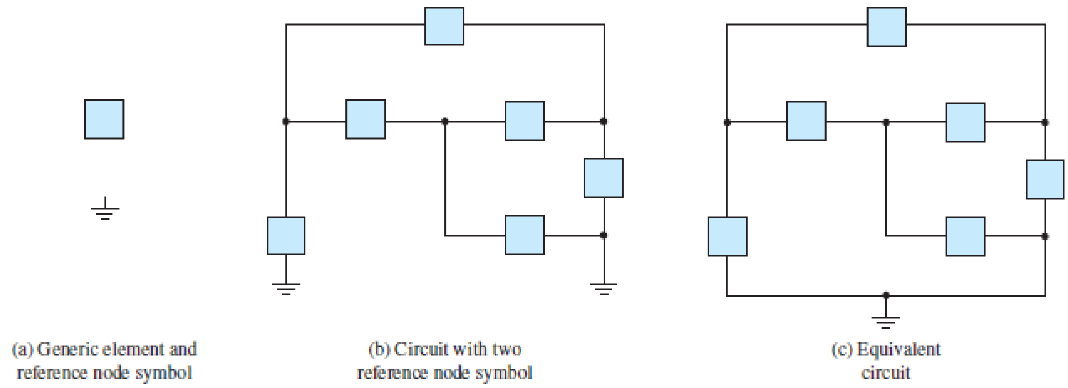


Figure 1.4 There can be one and only one reference node in a network although the reference node symbol may appear more than once in order to reduce the amount of displayed reference node wiring.

Branch

A **branch** is a single electrical pathway, consisting of wires and elements. A branch may contain one or more circuit elements as shown in Figure 1.5. By Page 8definition, the current *through* any one element in a branch is the same as the current through any other element in that branch; that is, there is one current in a branch, the *branch current.*

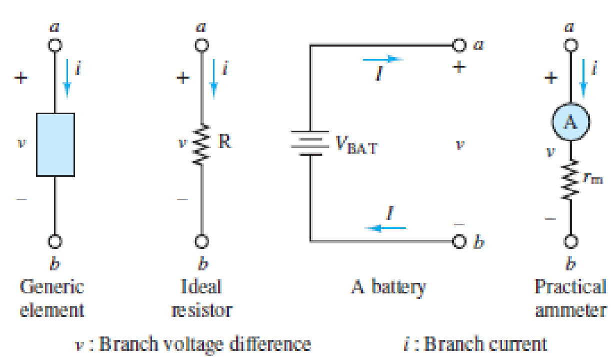


Figure 1.5 Examples of circuit branches

Elements that sit along the same branch are said to be in *series* and have the same current through them.

Loop

A **loop** is any closed pathway, physical or conceptual, as illustrated in Figure 1.6. Figure 1.6(a) shows that two different loops in the same circuit may share common elements and branches. It is interesting, and perhaps initially confusing, to note that a loop does not necessarily have to correspond to a closed electrical pathway, consisting of wires and elements. Figure 1.6(b) shows one example in which a loop passes directly from node *a* to node *c*.

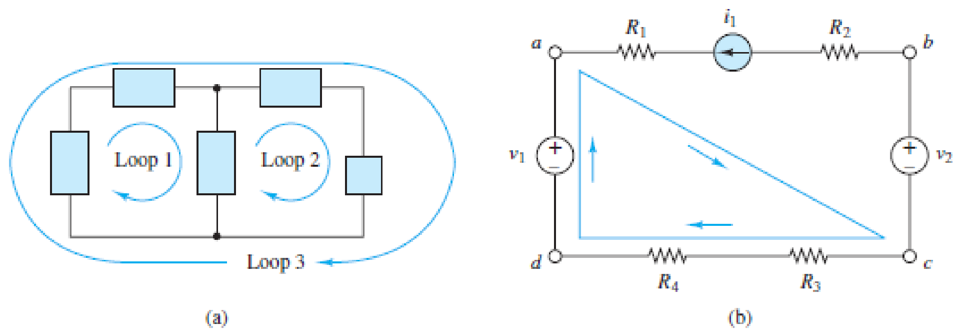


Figure 1.6 Examples of loops. How many nodes are in each of these circuits? [Answers: (a) 4; (b) 7]

Mesh

A **mesh** is a closed electrical pathway that does not contain other closed physical pathways. In Figure 1.6(a), loops 1 and 2 are meshes, but loop 3 is not a mesh because it encircles the other two loops. The circuit in Figure 1.6(b) has one mesh. Figure 1.7 illustrates how simple it is to visualize meshes.

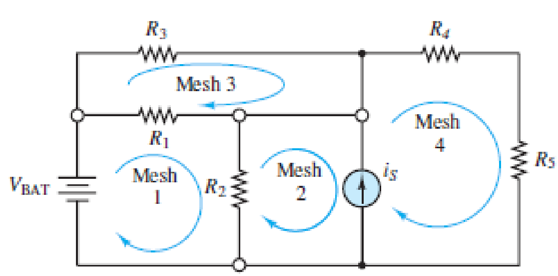


Figure 1.7 Circuit with four meshes. How many different closed electrical pathways are in this circuit? [Answer: 14]

EXAMPLE 1.1 Counting Nodes in a Network

Problem

Find the total number of nodes in each of the four networks of Figures 1.8–1.11.

Solution

Known Quantities: Wires and elements.

Find: The number of nodes in each network diagram in Figures 1.8 through 1.11

Schematics, Diagrams, Circuits, and Given Data: Figure 1.8 contains four elements: two resistors and two ideal voltage sources, one independent and one dependent. Figure 1.9 contains five elements: four resistors and one independent ideal current source. Figure 1.10 contains five elements: four resistors and one operational amplifier. Figure 1.11 contains three elements: two headlamps and one 12-V battery.

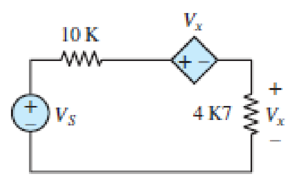


Figure 1.8

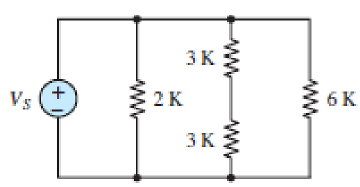


Figure 1.9

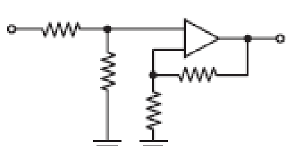


Figure 1.10

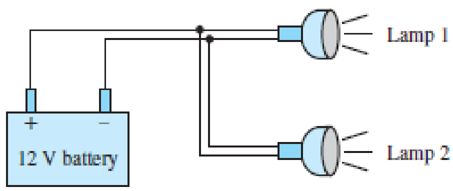


Figure 1.11

Assumptions: All wires are ideal.

Analysis: In Figure 1.8, all four elements are in a single electrical loop. There is one node between each pair of elements. Thus, there are *four nodes* in this network.

In Figure 1.9, all elements are connected between two nodes, one at the top, the other at the bottom of the circuit. In addition, there is a third node between the two 3-kΩ resistors. Thus, there are *three nodes* in this network.

In Figure 1.10, the nodes are expressly indicated by the black and white circles (note that the two circles on the far right denote the same node). In addition, the ground symbol repeated twice at the bottom of the circuit is also a node—the same node. Thus, there are *five nodes* in this network.

In Figure 1.11, there is one node between the positive + battery terminal and the two headlamps and one node between the negative − battery terminal and the two headlamps. Thus, there are *two nodes* in this network.

Comments: Notice that no knowledge of the elements is required to identify and count the nodes in a network.

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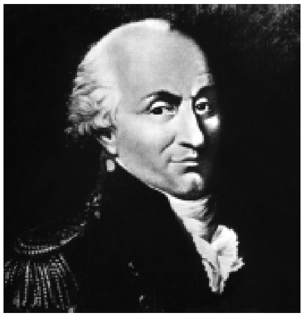
1.2 CHARGE CURRENT, AND KIRCHHOFF’S CURRENT LAW

The earliest accounts of electricity date from about 2,500 years ago, when it was discovered that static charge on a piece of amber was capable of attracting very light objects, such as feathers. The word *electricity* originated about 600 b.c.; it comes from *elektron*, which was the ancient Greek word for amber. The true nature of electricity was not understood until much later, however. Following the work of Alessandro Volta and his invention of the copper-zinc battery, it was determined that static electricity and the current in metal wires connected to a battery are due to the same fundamental mechanism: the atomic structure of matter, consisting of a nucleus—neutrons and protons—surrounded by electrons.



Charles Coulomb (1736–1806) (*INTERFOTO/Personalities/Alamy Stock Photo*)

The fundamental electric quantity is **charge.** The electron carries the smallest discrete unit of charge equal to:

$$q_e = -1.602 \times 10^{-19} \text{ C} \tag{1.1}$$

The amount of charge associated with an electron may seem rather small. However, the unit of charge, the **coulomb (C),** named after Charles Coulomb, is an appropriate unit for the definition of electric current since typical currents involve the flow of large numbers of charged particles. The charge of an electron is negative, by convention, to contrast it to the positive charge carried by a proton, which is the other charge-carrying particle in an atom. The charge of a proton is:

$$q_p = +1.602 \times 10^{-19} \text{ C} \tag{1.2}$$

Electrons and protons are often referred to as **elementary charges.**

Electric current is defined as the rate at which charge passes through a predetermined area, typically the cross-sectional area of a metal wire. Several other cases in which the current-carrying conduit is not a wire are explored later. Figure 1.12 depicts a macroscopic view of current i in a wire. With Δq units of charge flowing through the cross-sectional area A in Δt units of time, the resulting current i is defined by:

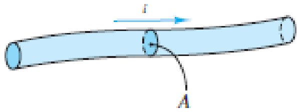


Figure 1.12 Current in an electric conductor is defined as the net flow rate of charge through the cross-sectional area A.

$$i \equiv \frac{\Delta q}{\Delta t} \qquad \frac{\text{C}}{\text{s}} \tag{1.3}$$

The arrow symbol associated with the current i is its *assumed* direction through the wire segment. A negative value for i would indicate that the actual direction is opposite the assumed direction. When large numbers of discrete charges cross A in a very small period, this relationship can be written in differential form as:

$$i \equiv \frac{dq}{dt} \qquad \frac{\text{C}}{\text{s}} \tag{1.4}$$

The unit of current is the **ampere,** where 1 ampere (A) = 1 coulomb/second (C/s). The name of the unit is a tribute to the French scientist André-Marie Ampère. The electrical engineering convention is that the direction of positive current is the direction of positive charge flow. This convention is sensible; however, it can be confusing at first since the mobile charge carriers in metallic conductors are, in fact, electrons from the *conduction band* of the metal. It may help to realize that when an electron travels in one direction the effect on the distribution of *net charge* Page 11is the same as if a proton had travelled in the opposite direction. In other words, positive current is used to represent the *relative* flow of positive charges.

Current in a Closed Path

Earlier in this chapter, a circuit was defined as "a complete and closed path around which a circulating electric current can flow." In fact, conservation of electric charge requires a closed path for any nonzero current.

To have a nonzero current, there must be a closed electrical path (i.e., a circuit).

For example, Figure 1.13 depicts a simple circuit, composed of a battery (e.g., a dry-cell or alkaline 1.5-V battery) and a lightbulb. Conservation of charge requires that the current i from the battery to the lightbulb is equal to the current from the lightbulb to the battery. No current (nor charge) is "lost" around the closed circuit. This principle was observed by the German scientist G. R. Kirchhoff[2] and is known as **Kirchhoff's current law (KCL).** This law states that *the net sum of the currents* *crossing any closed boundary (a node or supernode) must equal zero*. In mathematical terms:

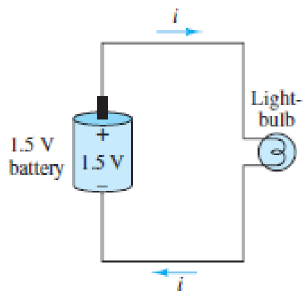


Figure 1.13 A simple electric circuit composed of a battery, a lightbulb, and two nodes

$$\sum_{n=1}^{N} i_n = 0 \qquad \text{Kirchhoff's current law (KCL)} \tag{1.5}$$

where the sign of currents entering the region surrounded by the closed boundary must be opposite to the sign of currents exiting the same region. In other words, the sum of currents "in" must equal the sum of currents "out." This statement leads to an alternate expression for KCL as:

$$\sum_{\text{in}} i = \sum_{\text{out}} i \qquad \text{Alternate KCL} \tag{1.6}$$

An application of Kirchhoff's current law is illustrated in Figure 1.14, where the simple circuit of Figure 1.13 has been augmented by the addition of two lightbulbs. One can find a relationship between the currents in the circuit by applying either version of KCL. To express the net sum of currents it is necessary to select a sign convention for currents entering and exiting a node. One possibility is to consider all currents entering a node as positive and all currents exiting a node as negative. (This particular sign convention is completely arbitrary.) The result of using this sign convention and applying the first version of KCL to node 1 is

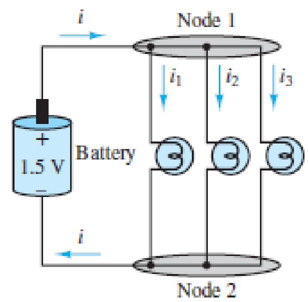


Figure 1.14 KCL appliedat node 1 results in$i - i_1 - i_2 - i_3 = 0$, or equivalently $i = i_1 + i_2 + i_3$.

$$i - i_1 - i_2 - i_3 = 0 \qquad \text{which is equivalent to} \qquad i = i_1 + i_2 + i_3$$

Note that the latter expression is exactly what would have been found if the alternate version of KCL had been applied. Also note that the result is the same if the opposite sign convention (i.e., currents entering and exiting the node are negative and positive, respectively) is used.

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EXAMPLE 1.2 Charge and Current in a Conductor

Problem

Find the total charge in a cylindrical conductor (solid wire) and compute the current through the wire.

Solution

Known Quantities: Conductor geometry, charge density, charge carrier velocity.

Find: Total charge of carriers Q; current in the wire I.

Schematics, Diagrams, Circuits, and Given Data:

Conductor length: $L = 1$ m.

Conductor diameter: $2r = 2 \times 10^{-3}$ m..

Charge density: $n = 10^{29}$ carriers/m^3.

Charge of one electron: $q_e = -1.602 \times 10^{-19}$.

Charge carrier velocity: $u = 19.9 \times 10^{-6}$ m/s.

Assumptions: None.

Analysis: To compute the total charge in the conductor, we first determine the volume of the conductor:

$$\text{Volume} = \text{length} \times \text{cross-sectional area}$$

$$\text{Vol} = L \times \pi r^2 = (1\text{ m})\left[\pi\left(\frac{2 \times 10^{-3}}{2}\right)^2 \text{m}^2\right] = \pi \times 10^{-6}\text{ m}^3$$

Next, we compute the number of carriers (electrons) in the conductor and the total charge:

$$\text{Number of carriers} = \text{volume} \times \text{carrier density}$$

$$N = \text{Vol} \times n = (\pi \times 10^{-6}\text{m}^3)\left(10^{29}\frac{\text{carriers}}{\text{m}^3}\right) = \pi \times 10^{23}\text{ carriers}$$

$$\text{Charge} = \text{number of carriers} \times \text{charge/carrier}$$

$$Q = N \times q_e = (\pi \times 10^{23}\text{ carriers}) \times \left(-1.602 \times 10^{-19}\frac{\text{C}}{\text{carrier}}\right) = -50.33 \times 10^3\text{ C}$$

To compute the current, we consider the velocity of the charge carriers and the charge density per unit length of the conductor:

$$\text{Current} = \text{carrier charge density per unit length} \times \text{carrier velocity}$$

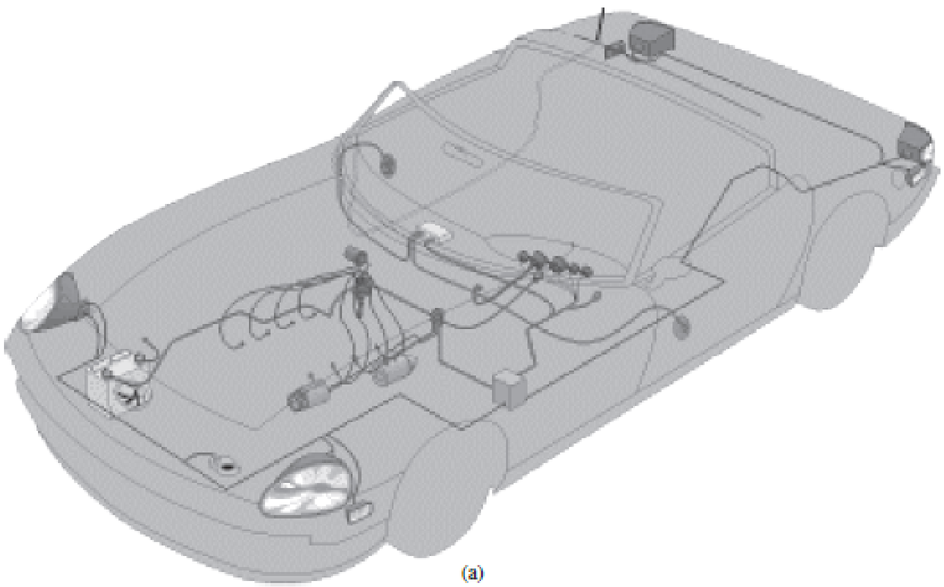
$$I = \left(\frac{Q}{L}\ \frac{\text{C}}{\text{m}}\right) \times \left(u\ \ \frac{\text{m}}{\text{s}}\right) = \left(-50.33 \times 10^3\ \frac{\text{C}}{\text{m}}\right)\left(19.9 \times 10^{-6}\ \frac{\text{m}}{\text{s}}\right) = -1\text{ A}$$

Comments: Charge carrier density is a function of material properties. Carrier velocity is a function of the applied electric field.

EXAMPLE 1.3 Kirchhoff's Current Law Applied to an AutomotiveElectrical Harness

Problem

Figure 1.15 shows an automotive battery connected to a variety of elements in an automobile. The elements include headlights, taillights, starter motor, fan, power locks, and dashboard panel. The battery must supply enough current to satisfy each of the “load” elements. Apply KCL to find a relationship between the currents in the circuit.



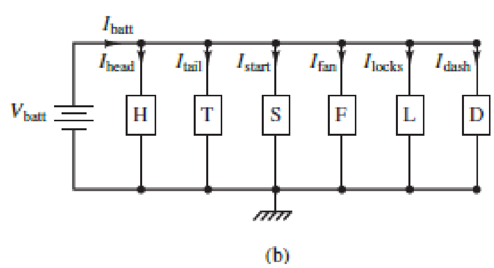


Figure 1.15 (a) Automotive electrical harness; (b) equivalent electric circuit diagram

Solution

Known Quantities: Components of electrical harness: headlights, taillights, starter motor, fan, power locks, and dashboard panel.

Find: Expression relating battery current to load currents.

Schematics, Diagrams, Circuits, and Given Data: Figure 1.15.

Assumptions: None.

Analysis: Figure 1.15(b) depicts the equivalent electric circuit, illustrating that the current supplied by the battery is divided among the various elements. The application of KCL to the upper node yields

$$I_{batt} - I_{head} - I_{tail} - I_{start} - I_{fan} - I_{locks} - I_{dash} = 0$$

or

$$I_{batt} = I_{head} + I_{tail} + I_{start} + I_{fan} + I_{locks} + I_{dash}$$

EXAMPLE 1.4 Application of KCL

Problem

Determine the unknown currents in the circuit of Figure 1.16.

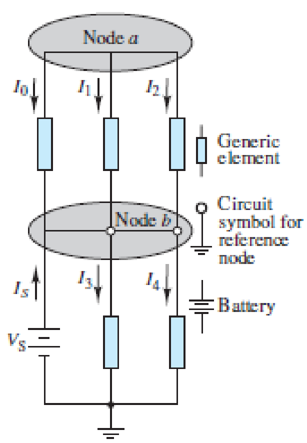


Figure 1.16 KCL yields$i_0 + i_1 + i_2 = 0$ at node a and $i_0 + i_1 + i_2 + i_S = i_3 + i_4$ atnode b.

Solution

Known Quantities:

$$I_S = 5\text{ A} \qquad I_1 = 2\text{ A} \qquad I_2 = -3\text{ A} \qquad I_3 = 1.5\text{ A}$$

Find: I_0 and I_4.

Analysis: Two nodes are clearly shown in Figure 1.16 as node *a* and node *b*; the third node in the circuit is the reference node. Apply KCL at each of the three nodes.

At node *a*:

$$I_0 + I_1 + I_2 = 0 \qquad \text{from} \qquad \sum i_{\text{out}} = \sum i_{\text{in}}$$
$$I_0 + 2 - 3 = 0$$
$$\therefore \qquad I_0 = 1 \text{ A}$$

Note that the assumed direction of all three currents is away from the node. However, I_2 has a negative value, which means that its actual direction is toward the node. The magnitude of I_2 is 3 A. The sign simply indicates direction relative to the assumed direction indicated in the diagram.

At node *b*:

$$I_0 + I_1 + I_2 + I_S = I_3 + I_4 \qquad \text{from} \qquad \sum i_{\text{in}} = \sum i_{\text{out}}$$
$$1 + 2 - 3 + 5 = 1.5 + I_4$$
$$\therefore \qquad I_4 = 3.5 \text{ A}$$

At the reference node: If we use the convention that currents entering a node are positive and currents exiting a node are negative, we obtain the following equations:

$$-I_S + I_3 + I_4 = 0$$
$$-5 + 1.5 + I_4 = 0$$
$$\therefore \qquad I_4 = 3.5 \text{ A}$$

Comments: The result obtained at the reference node is exactly the same as that calculated at node *b*. This fact suggests that some redundancy may result when we apply KCL at all nodes in a circuit. In Chapter 2 we develop a method called *node analysis* that ensures the derivation of the smallest possible set of independent equations.

EXAMPLE 1.5 Application of KCL

Problem

Apply KCL to the circuit of Figure 1.17, using the concept of a supernode to determine the source current i_{S1}.

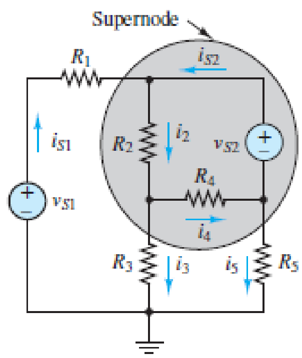


Figure 1.17 KCL applied at the boundary of the supernode yields $i_S1 = i_3 + i_5$.

Solution

Known Quantities:

$$i_3 = 2\text{ A} \qquad i_5 = 0\text{ A}$$

Find: i_{S1}.

Analysis: Apply KCL at the boundary of the supernode to obtain

$$i_{S1} = i_3 + i_5 \qquad \text{from} \qquad \sum i_{\text{in}} = \sum i_{\text{out}}$$
$$i_{S1} = 2 + 0 = 2\text{ A}$$

Comments: Notice that the same result for i_{S1} is obtained by applying KCL at the bottom node. This fact is another example of a redundant result that is sometimes obtained by applying KCL at two different nodes, including supernodes. When applied correctly, the *node analysis* method discussed in Chapter 2 prevents redundant equations.

CHECK YOUR UNDERSTANDING

Repeat the exercise of Example 1.4 when $I_0 = 0.5$ A, $I_2 = 2$ A, $I_3 = 7$ A, and $I_4 = -1$ A. Find I_1 and I_S.

Answer: $I_1 = -2.5$ A and $I_S = 6$ A

CHECK YOUR UNDERSTANDING

Use the result of Example 1.5 and the following data to compute the current i_{S2} in the circuit of Figure 1.17.

$i_2 = 3$ A $\quad i_4 = 1$ A

Answer: $i_{S2} = 1$ A

1.3 VOLTAGE AND KIRCHHOFF'S VOLTAGE LAW

Typically, work is required to move charge between two nodes in a circuit. The total *work per unit charge* is called **voltage**, and the unit of voltage is the **volt** in honor of Alessandro Volta.

$$1 \text{ volt (V)} = 1 \frac{\text{joule (J)}}{\text{coulomb (C)}} \tag{1.7}$$

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The voltage, or **potential difference,** across two nodes in a circuit is the energy (in joules) per unit charge (1 coulomb) needed to move charge from one node to the other. The direction, or *polarity*, of the voltage is related to whether energy is being gained or lost by the charge in the process.

Consider again the simple circuit of a battery and a lightbulb as shown in Figure 1.18. Experimental observations led Kirchhoff to formulate the second of his laws, **Kirchhoff's voltage law** (**KVL**), which states that that *the net change in electric potential around a closed loop is zero.* In mathematical terms:

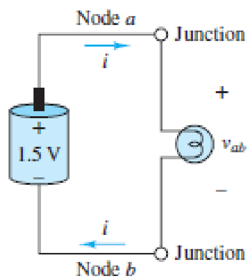
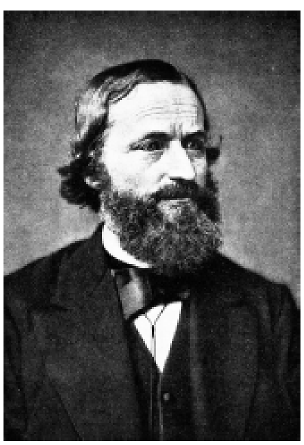


Figure 1.18 KVL applied clockwise from node b around the single loop circuit results in $1.5\text{ V} - v_{\text{ab}} = 0$, or equivalently $v_{\text{ab}} = 1.5\text{ V}$.

$$\sum_{n=1}^{N} v_n = 0 \qquad \text{Kirchhoff's voltage law} \tag{1.8}$$

Here, v_n are the changes in voltage from one node to another around a closed loop.



Gustav Robert Kirchhoff (1824–1887) (*bilwissedition Ltd. & Co. KG/Alamy Stock Photo)*

When summing these changes in voltage, it is necessary to account for the polarity of the change. Changes in voltage from the minus sign − to the plus sign + are considered positive (i.e., a rise in voltage), while those from plus to minus are considered negative (i.e., a drop in voltage). These two symbols act together to indicate the *assumed* direction of the change in voltage *across* an element, just as the arrow symbol is used to indicate the *assumed* direction of the current *through* an element or wire segment. A negative value indicates that the actual direction is opposite to the assumed direction.

An alternate but equivalent expression for KVL is that the sum of all voltage rises around a loop must equal the sum of all voltage drops around the same loop.

$$\sum_{\text{rises}} \upsilon = \sum_{\text{drops}} \upsilon \qquad \text{Alternate KVL} \tag{1.9}$$

In Figure 1.18, the *voltage across the lightbulb* is the change in voltage from node *a* to node *b*. This change can also be expressed as the difference between two node voltages, υ_a and υ_b. As stated earlier, the values of node voltages are relative to some reference node. Any single node may be chosen as the reference with its value set to zero, for simplicity, or any other convenient number. The circuit in Figure 1.18 has only two nodes, *a* and *b*, one of which can serve as the reference node. Select node *b* as the reference and set its value as $\upsilon_b = 0$. Then, observe that the battery's positive terminal is 1.5 V *above the reference*, so that $\upsilon_a = 1.5\text{ V}$. In general, the battery guarantees that node *a* will always be 1.5 V above node *b*. Mathematically, this fact is simply expressed as

$$\upsilon_a = \upsilon_b + 1.5\text{ V}$$
$$\upsilon_a = 1.5\text{ V} \qquad \text{when} \qquad \upsilon_b = 0 \text{ acts as the reference.}$$

The syntax used to express the *change* in voltage across the lightbulb, *from* node *b to* node *a*, is υ_{ab}, where

$$\upsilon_{ab} \equiv \upsilon_a - \upsilon_b = 1.5\text{ V}$$

This syntax is in accord with the + and − polarity indicator in that if υ_{ab} is positive, then $\upsilon_a > \upsilon_b$ and, in fact, node *a* is at a higher potential than node *b*, as suggested by the + and − syntax. It may be helpful to think of υ_{ab} as υ_a relative to node *b*.

Note that the work done in moving charge from node *a* to node *b* is directly proportional to the voltage across the lightbulb. Likewise, the work done moving Page 17charge back from *b* to *a* is directly proportional to the voltage across the battery. Let *Q* be the total charge that moves around the circuit per unit time, giving rise to current *i*. Then, the work *W* done *by* the battery *on Q*, from *b* to *a* (i.e., across the battery) is

$$W_{ab} = Q \times \upsilon_{ab} = Q \times 1.5\text{ V}, \qquad \text{work done by the battery on } Q$$

which is also equal to the work done *by Q on* the lightbulb, from *a* to *b* (i.e., across the lightbulb). One could express this work in the negative as the work done *by* the lightbulb *on Q*, from *a* to *b*.

$$W_{ba} = Q \times v_{ba} = -Q \times v_{ab} = -Q \times 1.5\ \text{V}$$

Note that the word *potential* is quite appropriate as a synonym of voltage, in that voltage is the potential energy per unit charge between two nodes in a circuit. If the lightbulb is disconnected from the circuit, a voltage v_{ab} still exists across the battery terminals, as illustrated in Figure 1.19. This voltage represents the ability of the battery to *supply* energy to the circuit. Likewise, the voltage across the lightbulb is associated with the work done by the lightbulb to *consume* or *dissipate* energy from the circuit. The rate at which charge is moved once a closed circuit is established depends upon the circuit element connected to the battery.

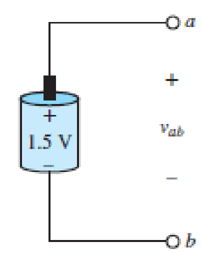


Figure 1.19 The voltage v_{ab} across the open terminals of the battery represents the potential energy available to move charge from *a* to *b* once a closed circuit is established.

The Reference Node and Ground

The concept of a reference node finds a practical use in the *ground* node of a circuit. Ground represents a specific, and usually clearly marked, reference node and voltage in a circuit. For example, the ground reference voltage can be identified with the enclosure or case of an instrument, or with the earth itself. In residential electric circuits, the ground reference is a large conductor, such as a copper spike or water pipe, that is buried in the earth. As mentioned, it is convenient and typical to assign the ground voltage reference a value of zero.

In practice, the term *ground* should not be applied to a node arbitrarily. However, the voltage value that is assigned to ground, while typically zero, is not consequential. A simple analogy with fluid flow illustrates this rule. Consider a tank of water, as shown in Figure 1.20, located at a certain height above the ground. The potential energy difference per unit mass due to gravity $u_{12} = g(h_1 - h_2)$ will cause water to flow out of the pipe at a certain flow rate. This quantity is completely analogous to the potential energy difference per unit charge $v_a - v_b$. Now assume that the height h_3 at ground level is chosen to be the zero potential energy Page

18reference. Is the flow of water in the pipe changed due to this choice? Of course not. Is the flow of water in the pipe dependent upon the height $h_2 - h_3$ of the support structure? Again, the answer is no. The truth of these statements is demonstrated by rewriting the *head* of the water tank $h_1 - h_2$ as $(h_1 - h_3) - (h_2 - h_3)$ and by noting that the potential energy difference per unit mass can be written as the difference in potential energy per unit mass relative to the ground. That is,

Figure 1.20 An analogy between water flow and electric current illustrates the relation between potential differences and a ground reference potential.

$$u_{12} = g(h_1 - h_2) = [g(h_1 - h_3)] - [g(h_2 - h_3)] = u_{13} - u_{23}$$

Note that the values of u_{13} and u_{23} depend upon the value assigned to h_3; however, the value of u_{12} does *not* depend upon the value assigned to h_3. If this result were not true, our experience of the physical world would be very strange indeed, and not only because the flow of water from a tank would depend significantly upon the height of the tank itself. It is the relative difference in potential energy that matters in the water tank problem. So it is with electric circuits. The current through an element depends upon the potential difference (i.e., voltage) *across* the element and not on the selection of a reference node nor the arbitrary value of the ground reference node.

Another familiar scenario is that of a skydiver leaping from an airplane and parachuting to the surface below (see Figure 1.21). To quantify the potential energy U of the skydiver it is first necessary to choose a reference height h_0 such that$U = mg\Delta h = mg(h - h\ 0)$, where h represents the position of the skydiver. One possible reference is the height of the airplane such that the potential energy of the skydiver is negative ($U < 0$). However, that reference is not particularly meaningful. The surface of the earth is a more meaningful reference to the skydiver, who knows that a soft landing depends upon dissipating most of the initial potential energy through collisions with air molecules rather than through a collision with the surface. The skydiver knows that her fate is unchanged by her choice of reference; however, some choices are more meaningful than others. So it often is with electric circuits.